LAMINAR MIXED CONVECTION IN A DOUBLE LID-DRIVEN SQUARE CAVITY

Maximilian Serguei Mesquita, engmaxmesquita@yahoo.com.br
Departamento de Engenharias e Computação – DECOM
Universidade Federal do Espírito Santo – UFES/CEUNES
29.933-415 São Mateus, ES, Brasil

Luzia Aparecida Tofaneli , ltofa@hotmail.com
SENAI - Departamento Regional da Bahia, Centro Integrado de Manufatura e Tecnologia
41650-010 - Salvador, BA, Brasil

Abstract. *The present work investigated the configuration of steady two-dimensional flow accompanied by heat and mass simultaneous transport in a lid-driven cavity with a moving heated lid and a moving cooled lid were investigated numerically in a process engineering context of drying. Governing equations are solved using the finite volume method and the algebraic equation set is relaxed with the SIP procedure. In a parameter study for horizontal and vertical orientation of the cavity, the dependence of heat and mass transfers rates on the velocities of the walls and on the species concentration boundary conditions was investigated.*

Keywords*: Lid Driven cavity; Two dimensional; incompressible flow; heat and mass transfer*

1. INTRODUCTION

The problem on laminar mixed convection with lid driven flows has multiples applications in the field of thermal engineering. Such problems are of great interest, for example in electronic device cooling, high-performance building insulation , multi shield structures used for nuclear reactors , food proceeding , glass production , solar power collectors , furnace, drying technologies , etc .

The study of double-diffusive natural convection in porous media has many environmental and industrial applications, including grain storage and drying, petrochemical processes, oil and gas extraction, contaminant dispersion in underground water reservoirs, electrochemical processes, etc [Mamou , Vasseur and Bilgen (1995), Mohamad and Bennacer (2002), Goyeau, Songbe and Gobin (1996), Nithiarasu, Sundararajan and Seetharamu (1997), Mamou et al (1998), Bennacer et al (2001) and Bennacer, Beji and Mohamad (2003)]. In some specific applications, the fluid mixture may become turbulent and difficulties arise in the proper mathematical modeling of the transport processes under both temperature and concentration gradients.

Modeling of macroscopic transport for incompressible flows in rigid porous media has been based on the volume-average methodology for either heat [Hsu and Cheng (1990)] or mass transfer [Bear (1972), Bear and Batchmat (1967),Whitaker (1966), Whitaker (1967), Whitaker (1969)]. If time fluctuations of the flow properties are considered, in addition to spatial deviations, there are two possible methodologies to follow in order to obtain macroscopic equations: a) application of time-average operator followed by volume-averaging [Kuwahara and Nakayama (1998) and Nakayama and Kuwahara (1999)], or b) use of volume-averaging before time-averaging is applied [Lee and Howell (1987), Wang and Takle (1995), Another and Lage (1997) and Getchewa , Minkowycz and Lage (2000)]. This work intends to present a set of macroscopic mass transport equations derived under the recently established double decomposition concept [Pedras and De Lemos (2000), Pedras and De Lemos (2001), Pedras and De Lemos (2001a), Pedras and De Lemos (2001b)], through which the connection between the two paths a) and b) above is unveiled. That methodology, initially developed for the flow variables, has been extended to heat transfer in porous media where both time fluctuations and spatial deviations were considered for velocity and temperature [Rocamora and De Lemos (2000)]. Buoyant flows [De Lemos and Braga (2003)] and mass transfer [De Lemos and Mesquita (2003)] have also been investigated. Recently, a general classification of all proposed models for turbulent flow and heat transfer in porous media has been published [De Lemos and Pedras (2001), De Lemos and Tofaneli (2004)].

Heat and simultaneous mass transfer in lid-driven enclosures have received less attention in the literature [Khanafer and Vafai (2002)]. In drying technology, better understanding of drying processes is vital for optimum performance of drying chamber. In [Alleborn , razzillier and Durst (1999)] investigated a two-dimensional flow accompanied by heat and mass transport in a shallow lid-driven cavity with moving heated and a moving cooled lid. Their results showed the drying rates were enhanced by increasing the web velocity and become increasingly independent of the gravity orientation because of the dominance of forced convection.

The present study is focused on the numerical analysis of heat and mass simultaneous transfer in a square enclosure using the generalized numerical model of the momentum, energy and chemical species transport equations. A wide range of parameters such as the Reynolds number, Richardson number, Prandtl number, Lewis number, Grashof number, Buoyancy ratio is considered in the present research to show the significance and influences of these parameters on the heat and mass transfer phenomena.

2. PROBLEM DEFINITION AND GOVERNING EQUATIONS

Consider a two-dimensional enclosure of height H and width L filled a fluid as shown in Fig.1. The effect of heat conduction in the solid walls is assumed be negligible. The vertical walls are assumed to be insulated, adiabatic, and impermeable to mass transfer. The fluid in the enclosure is Newtonian, incompressible and laminar. The effects of Soret (thermal diffusion) and Dufour (diffusion thermo) are neglected in the present study. The top and bottom wall, moving at constant velocity U_0 is kept at high temperature and mass concentration (T_H and C_H) while the horizontal wall is kept at low temperature and concentration (T_L and C_L) . The thermo physical properties of the fluid are assumed to be constant except the density variation in the buoyancy force, which is approximated according the Boussinesq hypothesis. This variation, due both temperature and mass concentration gradients, can be described by the following equation:

$$\rho = \rho_0\left[1-\beta_T\left(T-T_{ref}\right)-\beta_C\left(C-C_{ref}\right)\right] \tag{1}$$

Where β_T and β_C are the coefficients for thermal and concentration expansions, respectively:

$$\beta_T = -\frac{1}{\rho_0}\left(\frac{\partial\rho}{\partial T}\right)_{P,C} , \quad \beta_C = -\frac{1}{\rho_0}\left(\frac{\partial\rho}{\partial C}\right)_{P,T} \tag{2}$$

The continuity, momentum, energy and mass concentration of chemical species equations can be written as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3}$$

$$u\frac{\partial u}{\partial x}+V\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\nabla^2 u \tag{4}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\nabla^2 v+g\left[\beta_T\left(T-T_{ref}\right)\right] \tag{5}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\nabla^2 T \tag{6}$$

$$u\frac{\partial C_\ell}{\partial x}+v\frac{\partial C_\ell}{\partial y}=D_\ell\nabla^2 C_\ell \tag{7}$$

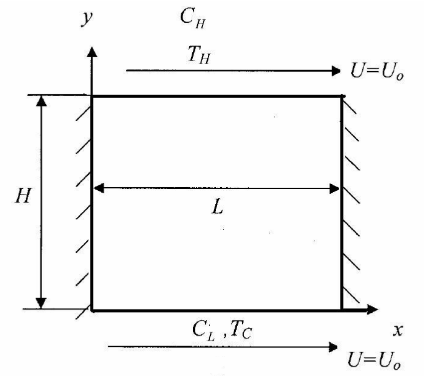
Where u and v are the velocity components in x and y directions respectively, ρ is the density of the fluid, p is the total pressure and ν is the kinematic viscosity of the fluid. The gravity acceleration is defined by $\mathbf{g}$. T, C_ℓ, T_{ref} and C_{ref} are the temperature, mass concentration , reference temperature and the reference mass concentration, respectively, α is the thermal diffusivity and D_ℓ is the mass diffusion coefficient. The transport dimensionless parameters, the solutal Grashoff (Gr_C), the thermal Grashoff (Gr_T), the Prandtl ($\Pr$), Reynolds number (Re), Schmidt number (Sc), the buoyancy ratio (N), Lewis number (Le), Richardson number (Ri) and the Rayleigh number (Ra) are given by:

$$\mathrm{Re}=\frac{U_0 H}{\nu} , \quad Gr_C=\frac{g\beta_C\left(C_H-C_{ref}\right)H^3}{\nu^2} , \quad Gr_T=\frac{g\beta_T\left(T_H-T_{ref}\right)H^3}{\nu^2} \tag{8}$$

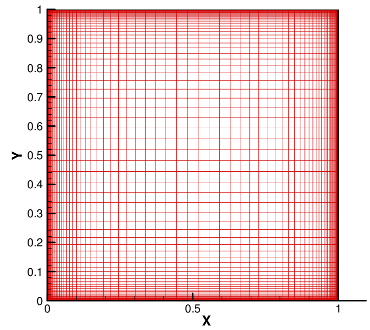
$$\Pr=\frac{\nu}{\alpha},\ Sc=\frac{\nu}{D_\ell},\ Le=\frac{\alpha}{D_\ell},\ N=\frac{\beta_C\Delta C}{\beta_T\Delta T}=\frac{Gr_C}{Gr_T},\ Ri=\frac{Gr_T}{\mathrm{Re}^2}\ \text{and}\ Ra=Gr_T\cdot\Pr \tag{9}$$

The aspect ratio of enclosure is $A = L/H$. The average heat and mass fluxes at walls are representing in dimensionless forms by the average Nusselt and Sherwood numbers:

$$Nu = \frac{1}{A}\int_0^A \left(\frac{\partial T}{\partial y}\right)_{y=0} dx \text{ and } Sh = \frac{1}{A}\int_0^A \left(\frac{\partial C}{\partial y}\right)_{y=0} dx \tag{10}$$



a)



b)

Figure 1 –Sketch of the cavity and computational grid

3. NUMERICAL METHOD

The numerical method employed for discretizing the governing equations in the control-volume approach. The flux blended deferred correction which combines linearly the Upwind Differencing Scheme (UDS) and Central Differencing Scheme (CDS), was used for interpolating the convective fluxes. The well-established SIMPLE algorithm is followed for handling the pressure-velocity coupling. Individual algebraic equation sets were solved by the SIP procedure. Details on the validation of the numerical tools, optimization and checking of numerical grids here employed can be found in previous publications, which include buoyant and forced flows in porous , hybrid and clean media for different geometries and boundary conditions, for more details see refers [Braga and De Lemos (2003), Braga and De Lemos (2005), Mesquita and De Lemos (2004), Mesquita and De Lemos (2004a) and Saito and De Lemos (2006)].

4. DISCUSSION AND RESULTS

The numerical code used in the present investigation has been used to carry out a number of simulations for a wide range of controlling parameters such as buoyancy ratio (N), Reynolds number (Re) , Richardson number (Ri) and Lewis number (Le). The results are presents for three different heat transfer regimes: the pure forced convection, the pure natural convection and the mixed convection. Each regime is given in a separate subtitle in the following:

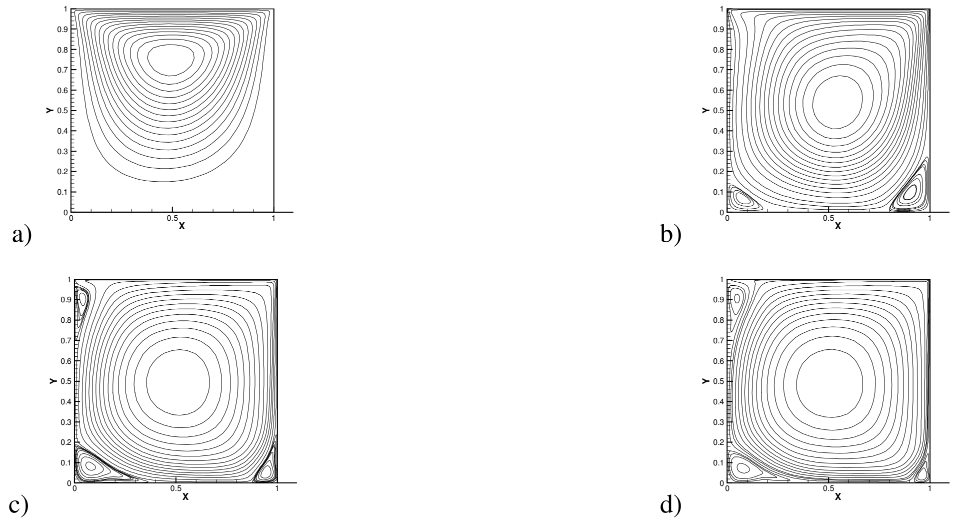


Figure 2 - Visualization from streamlines fields for lateral shear driven cavity: a) $\mathrm{Re}=10^{+02}$, b) $\mathrm{Re}=10^{+03}$, c) $\mathrm{Re}=5\times10^{+03}$ and d) $\mathrm{Re}=10^{+04}$.

Forced convection - Figure 2 shows the flow in a horizontal shear-driven cavity (occasioned by upper moving wall) for $\mathrm{Re}=10^{+02},10^{+03},5\times10^{+03}$ and 10^{+04} respectively. These results are in close agreement with [Guia et al. (1982), Sivaloganathan and Shaw (1988)]. Fig. 2 show the growth in the secondary vortex at bottom right-hand corner that is expected with an increase in Reynolds number. There is clear loss of symmetric of the recirculation and the primary vortex centre has moved off the vertical mid-cavity line as discussed in [Sivaloganathan and Shaw (1988)].

Natural convection - For the natural convection of air in a differentially-heated cavity, the flow and temperatures fields are given in Figure 3. The left wall of the cavity is kept at temperature of T_H , while the right wall being kept at a temperature of T_C lower than T_H . The horizontal walls are kept as adiabatic. Although a wide range of Rayleigh number (Ra) was considered, just three representatives examples, for $Ra=10^{+03},10^{+04}$ and 10^{+05} were given here due to the limitation of this paper space. For this geometry [De Vahl Davis (1983)] obtained bench-mark solutions, which serves as a comparison tool for many studies. The present results correspondence very well with those of [De Vahl Davis (1983)]. At $Ra=10^{+03}$ the streamlines in Fig 3 a). indicates the existence of a single vortex with center in the middle of the cavity. Corresponding isotherms in the Fig. 3 are almost parallel to the heated walls, indicating that most of the heat transfer is transferred by conduction. The vortex is generated due the horizontal temperature gradient across the section. This gradient, $\partial T/\partial x$ is negative everywhere, inducing a clockwise oriented vorticity. When the Rayleigh number is increased to $Ra=10^{+04}$, Fig. 3 b) the central vortex is distorted into an elliptic shape and the effect of convection is more pronounced in the isotherms, Fig. 3. Temperature gradients are stronger near the vertical walls, but decrease in the center region. For $Ra=10^5$, Fig. 3 c), the behavior continues. The central vortex is elongated and two secondary vortices appear inside it. Heat transfer by convection in the viscous boundary layer alters the temperature distribution to such an extent that temperature gradients in the center of the domain are close to zero. Fig. 3c) also show that, with this change in the sign of the source term, negative vorticity is induced within the domain. This also causes the development of secondary vortices in the core.

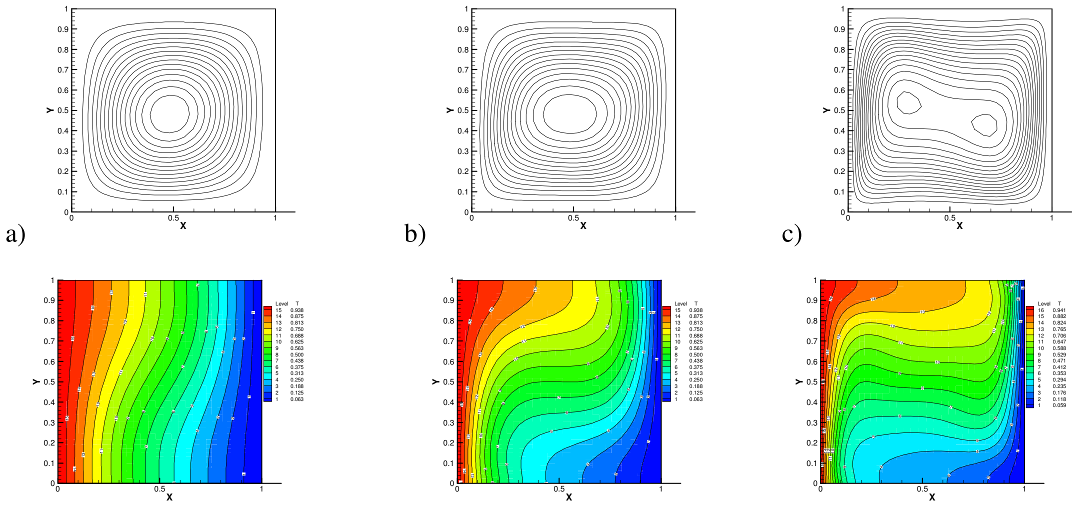


Figure 3 – Streamlines and Isotherms for Natural Convection case at $Ra = 10^{+03}, 10^{+04}$ and 10^{+05}

Mixed convection – It has been already explained, two different heat transfer regime and consequently more than three different thermal boundary conditions considered here, result in two different heat and mass transfer mechanisms: Buoyancy – aiding and opposing. It was shown in Fig. 4 (for more details see reference [Aydan (1999)], the movement of left wall was leading a clockwise rotating cell. The hot left and cold right walls, is considered, the resulting buoyancy induce a clockwise rotating flow. Such flows reinforce the recirculation cell driven by the moving of left wall. In this case, the buoyancy has an aiding effect on the forced convection. For the buoyancy aiding case, explained above, the flow and temperatures fields in terms of the numerical streamlines and isotherms for several values of mixed convection parameter, Richardson number (Ri) covering a) $Ri = 0.01$, b) $Ri = 0.1$, c) $Ri = 5$ and $Ri = 100$ are considered. Since the Reynolds number is kept constant at $\mathrm{Re} = 100$. As seen from Fig. 4 , an increase of Ri from 0.01 to 0.1 does not make any significance change in the flow and temperature fields. In the other words, the predominance of the forced convection over natural convection maintains because the buoyancy effects is still weak to affect the flow pattern. The buoyancy becomes progressively a stronger role by the increasing of the Ri, at $Ri \geq 5$ the hydrodynamic and thermals fields makes the shear and buoyancy effect, respectively, important and characteristic. In this case, the natural convection becomes dominant mechanism with the negligible forced convection.

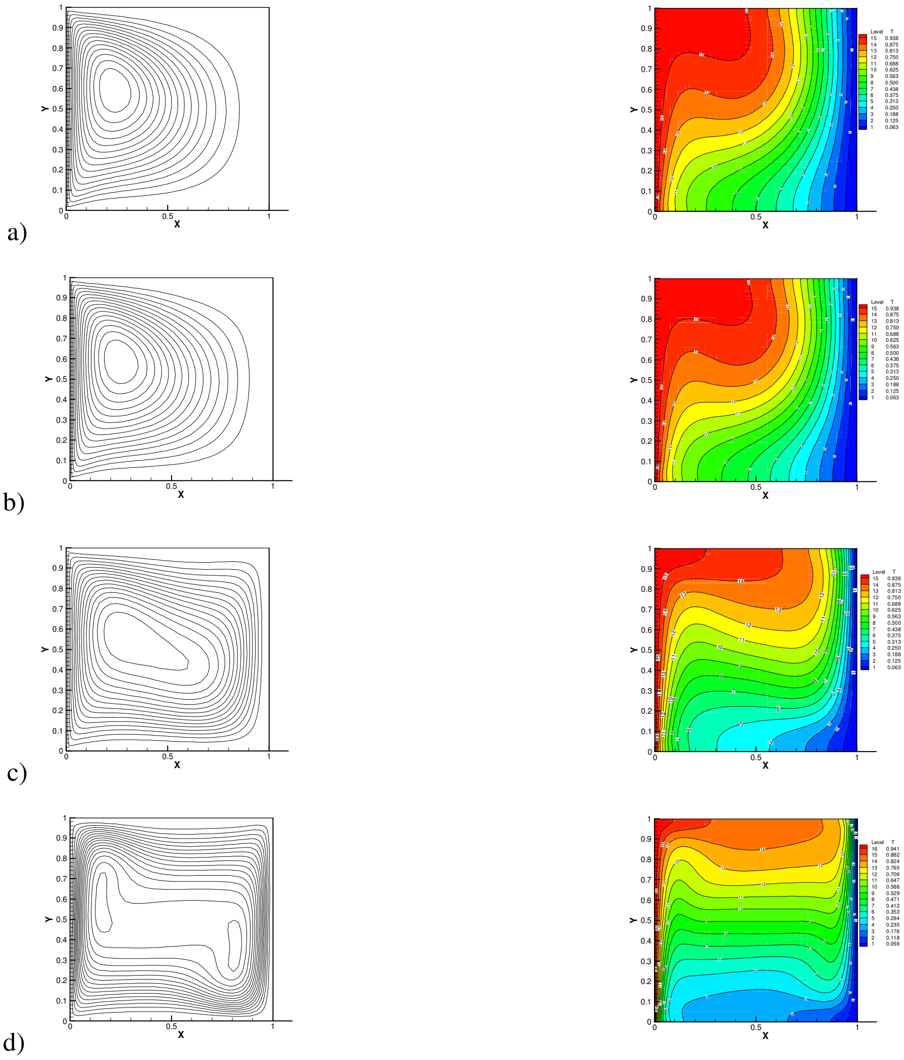


Figure 4 - Streamlines and isotherms at different values of Richardson number for aiding-buoyancy flow: a) $Ri = 0.01$, b) $Ri = 0.1$, c) $Ri = 5$ and $Ri = 100$

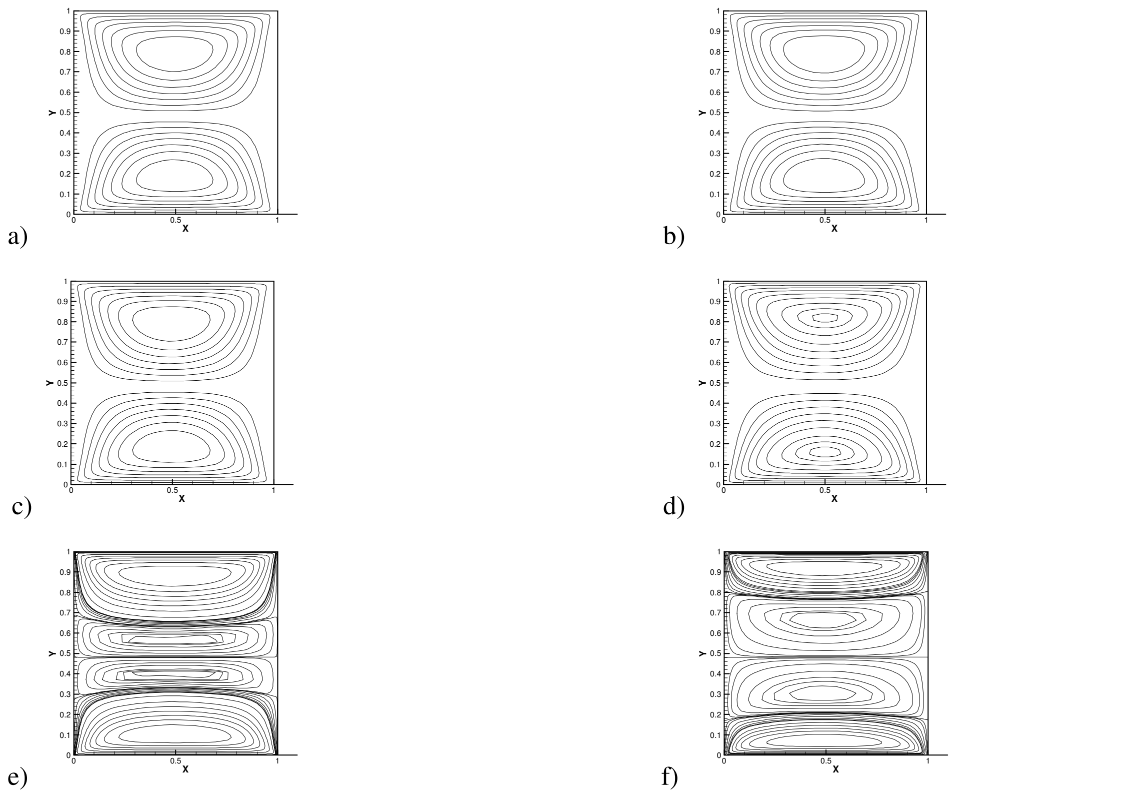


Figure 5 - Streamlines for different combinations of Richardson number with Re = 100: a) Ri = 0.001, b) Ri = 0.01, c) Ri=0.1, d) Ri=1.0, e) Ri=10 and f) Ri=100

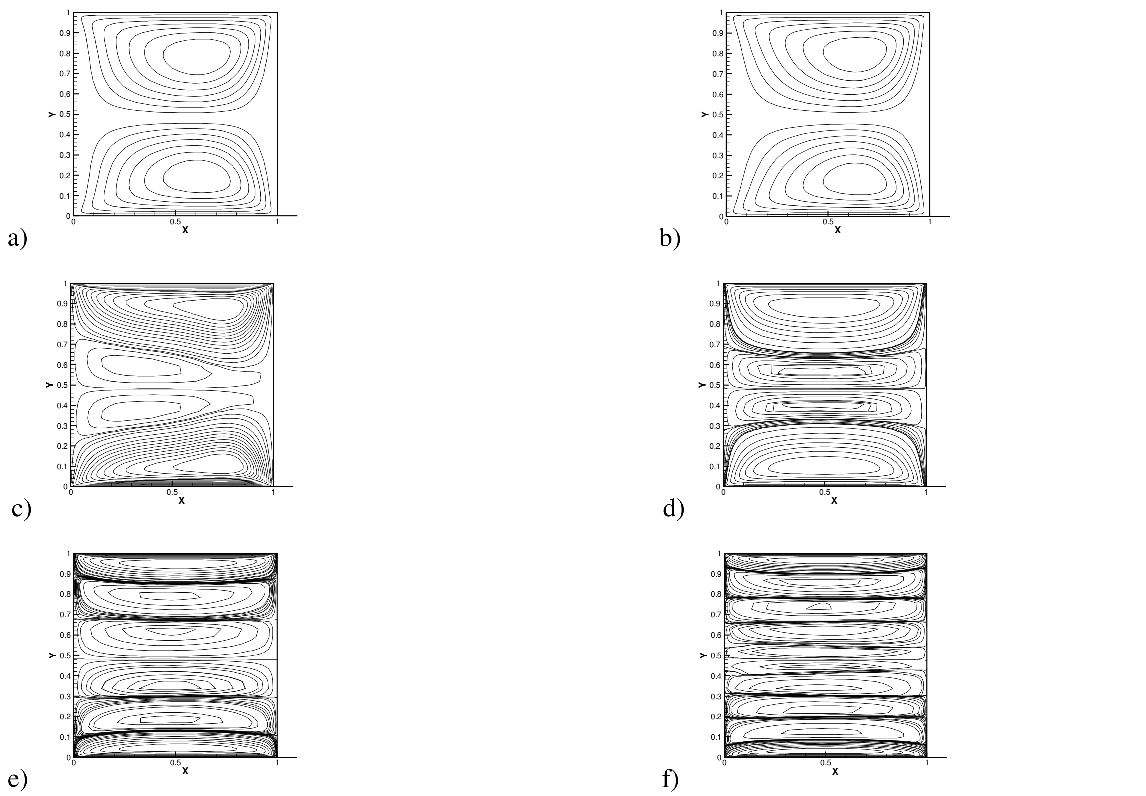


Figure 6 - Streamlines for different combinations of Richardson number with Re = 400 : a) Ri = 0.001 , b) Ri= 0.1 , c) Ri = 1 , d) Ri=10, e) Ri=100 and f) Ri=1000

3. ACKNOWLEDGEMENTS

The authors are indebted to CNPQ and CAPES, Brazil, for their invaluable support during the course of this research endeavor.

4. REFERENCES

[1] Alleborn, N., Raszillier, H., Durst, F., Lid-Driven Cavity with Heat and Mass Transport, Int. Journal Heat Mass Transfer, Vol. 42, pp. 833-853, 1999.
[2] Antohe, B. V. and Lage,J. L., A general two-equation macroscopic turbulence model for incompressible flow in porous media, International Journal Heat Mass Transfer, 40 (13), (1997), 3013 - 3024.
[3] Aydain, O., Aiding and Opposing Mechanisms of Mixed Convection in a Shear and Buoyancy-Driven Cavity, Int. Comm. Heat Mass Transfer, Vol. 26 No. 7, pp. 1019-1028, 1999.
[4] Bear, J., Dynamics of Fluids in Porous Media, Dover, New York (1972).
[5] Bear, J., and Bachmat, Y., A generalized theory on hydrodynamic dispersion in porous media, I.A.S.H. Symp. Artificial Recharge and Management of Aquifers, Haifa, Israel, P.N. 72, pp. 7-16, I.A.S.H. (1967).
[6] Braga, E.J., De Lemos, M.J.S., Int. Journal Heat Transfer 32 (2005) 1289 – 1297.
[7] de Lemos , M. J. S. and Braga, E. J. , Modeling of turbulent natural convection in porous media, International Communications Heat Mass Transfer, 30 (5), (2003), 615 - 624.
[8] de Lemos, M. J. S. and Mesquita, M. S. , Turbulent mass transport in saturated rigid porous media, International Communications Heat Mass Transfer, 30, (2003), 105 - 113.
[9] de Lemos, M. J. S. and Pedras, M. H. J., Recent mathematical models for turbulent flow in saturated rigid porous media, Journal of Fluids Engineering, 123 (4), (2001), 935 - 940.
[10] de Lemos, M.J.S. and Tofaneli, L. A., Modeling of double diffusive turbulent natural convection in porous media, International Journal Heat Mass Transfer, 47, (2004), 4233-4241.
[11] De Vahl Davis, G. and J. P. Jones, Int. Journal Numerical Methods Fluids 13, 227 (1983)
[12] Getachewa,D., Minkowycz, W.J. and Lage,J. L., A modified form of the kappa-epsilon model for turbulent flows of an incompressible fluid in porous media, International Journal Heat Mass Transfer, 43 (16), (2000), 2909 - 2915.
[13] Ghia, U., Guia, K.N., Shin, C.T., High-Re Solutions for Incompressible Flow Using the Navier-Stokes Equations and a Multigrid Method, Journal of Computational Physics, 48 1982 387-411.
[14] Goyeau, B., Songbe, J. P. and Gobin, D., Numerical study of double-diffusive natural convection in a porous cavity using the Darcy-Brinkman formulation, International Journal of Heat and Mass Transfer, 39 (7), (1996), 1363-1378.
[15] Gray, W. G. and Lee, P. C. Y. , On the theorems for local volume averaging of multiphase system, Int. J. Multiphase Flow, 3, (1977), 333 -340.
[16] Hsu, C.T. and Cheng, P., Thermal dispersion in a porous medium, International Journal Heat Mass Transfer, 33, (1990), 1587-1597.
[17] Lee, K. and Howell, J. R., Forced convective and radiative transfer within a highly porous layer exposed to a turbulent external flow field, Proceedings of the 1987 ASME-JSME Thermal Engineering Joint Conf., Honolulu, Hawaii, vol. 2, 377-386, ASME, New York, N.Y. (1987).
[18] Khanafer, K., Vafai, K., Double-Diffusive Mixed Convection in a Lid-Driven Enclosure Filled with a Fluid-Saturaded Porous Medium, Numerical Heat Transfer, Part A, 42: 465-486, 2002.
[19] Kuwahara, F., Nakayama, A., and Koyama, H., A numerical study of thermal dispersion in porous media, Journal of Heat Transfer, 118, (1996), 756.
[20] Kuwahara, F., and Nakayama, A., Numerical modeling of non-Darcy convective flow in a porous medium, Heat Transfer 1998: Proc. 11th Int. Heat Transf. Conf., Kyongyu, Korea, vol. 4, pp. 411-416, Taylor & Francis Washington, D.C. (1998).
[21] Mamou, M., Vasseur, P. and Bilgen, E., Multiple solutions for double-diffusive convection in a vertical porous enclosure, International Journal of Heat and Mass Transfer, 38 (10), (1995), 1787-1798.
[22] Mamou, M., Hasnaoui, M., Amahmid, A., and Vasseur, P., Stability analysis of double diffusive convection in a vertical brinkman porous enclosure, International Communications in Heat and Mass Transfer, 25 (4), (1998), 491-500.
[23] Merrikh, A., A.; Lage, J. L., Effect of Distributing a Fixed Amount of Solid Constituent inside a Porous Medium Enclusure on the Heat Transfer Process, ICAPM 2004 – Proceedings of Intern. Conference Applications of Porous Media, 2004, p.51.
[24] Mesquita, M.S., de Lemos, M.J.S., Mass Dispersion coefficients for turbulent flow in a infinite porous medium, In proceedings of ASME Heat Transfer / Fluids Engineering Summer Conference 2004, HT / FED 2004 1, pp. 561-568.

[25] Mesquita, M.S., de Lemos, M.J.S., Macroscopic modeling of turbulent transport in heterogeneous porous media, 2004a, ASME-Heat Transfer Division (Publication) HTD 375 (1) art. No. IMECE2004-62405, pp. 365-373.
[26] Mesquita, M.S., de Lemos, Mixed convection in square vented enclosure filled with a porous material using the multigrid method, 2007 in proceedings of AIChE Annual Meeting.

[27]Mohamad, A. A., and Bennacer, R., Double diffusion natural convection in an enclosure filled with saturated porous medium subjected to cross gradients; stably stratified fluid, International Journal of Heat and Mass Transfer, 45 (18), (2002), 3725-3740.
[28] Nakayama and Kuwahara, F., A macroscopic turbulence model for flow in a porous medium, Journal of Fluids Engineering, 121, (1999), 427 - 433.
[29] Nithiarasu, P., Sundararajan, T., Seetharamu, S. N., Double-diffusive natural convection in a fluid saturated porous cavity with a freely convecting wall, International Communications in Heat and Mass Transfer, 24 (8), (1997), 1121-1130.
[30] Pedras, M.H.J. and de Lemos, M.J.S. , On the definition of turbulent kinetic energy for flow in porous media, Internernational Communications Heat and Mass Transfer, 27 (2), (2000), 211 - 220.
[31] Pedras, M.H.J. and de Lemos, M.J.S., Macroscopic turbulence modeling for incompressible flow through undeformable porous media, International Journal Heat and Mass Transfer, 44 (6), (2001), 1081 - 1093.
[32] Pedras, M.H.J. and de Lemos, M.J.S. , Simulation of turbulent flow in porous media using a spatially periodic array and a lowre two-equation closure, Numerical Heat Transfer - Part A Applications, 39 (1), (2001a), 35.
[33] Pedras, M.H.J and de Lemos, M.J.S. , On the mathematical description and simulation of turbulent flow in a porous medium formed by an array of elliptic rods, Journal of Fluids Engineering, 23 (4), (2001b), 941 - 947.
[34] Rocamora Jr., F.D. and de Lemos, M.J.S., Analysis of convective heat transfer for turbulent flow in saturated porous media, International Communications Heat and Mass Transfer, 27 (6), (2000), 825 - 834.
[35] Saito ,M.B., de Lemos, M.J.S., Interfacial heat transfer coefficient for turbulent flow over an array of square rods, Int. J. Heat Mass Transfer 128 (2006) 444-452.
[36] Sivaloganathan, S., Shaw, G. J., A Multigrid Method for Recirculating Flows, International Journal for Numerical Methods in Fluids, Vol. 8 1988 417- 440.
[37] Slattery , J.C., Flow of viscoelastic fluids through porous media, A.I.Ch.E. J., 13, (1967) 1066 - 1071.
[38] Wang , H. and Takle , E.S., Boundary-layer flow and turbulence near porous obstacles .1. derivation of a general equation set for a porous-medium, Boundary Layer Meteorology, 74, (1995), 73 - 78.
[39] Whitaker, S., Advances in theory of fluid motion in porous media, Indust. Eng. Chem., 61, (1969), 14 - 28.
[40] Whitaker, S., Equations of motion in porous media, Chem. Eng. Sci., 21, (1966), 291.
[41] Whitaker, S., Diffusion and dispersion in porous media, J. Amer. Inst. Chem. Eng, 3 (13), (1967), 420.
[42] Trevisan , O. and Bejan, A. Natural convection with combined heat and mass transfer buoyancy effects in a porous medium, International Journal Heat and Mass Transfer, 28, (1985), 1597-1611.

5. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.