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Classical and fractal analysis of vehicle demand on the ferry-boat system

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ABSTRACT

Transportation problems are important complex systems because of the increased number of vehicles in cities. In this paper, we study time series of vehicle demand by using the ferry-boat system between Salvador city and Itaparica island, in Bahia, Brazil. We compare the traditional demand analysis (ARIMA method) with the self-affine ones (the scaling exponent α and the density of crossing points ρ). In addition, taking into account the inherent self-affine behavior we study the stationary states of this dynamic process by using a nonlinear Fokker–Planck equation. The present findings indicate that the scaling exponent α describes some properties of flux of vehicles using the ferry-boat system. The behavior of α gives an alternative explanation about demand analysis, and the nonlinear Fokker–Planck equation presents a solution close to the stationary behavior of this complex dynamical analysis.

1. Introduction

Transportation problems have attracted much attention in the field of physics [1–5] and have been studied from a perspective of statistical mechanics and nonlinear dynamics [6–11]. Maritime transport is one of these problems, mainly in cities with a great expanses of water, such as, Salvador, Bahia (Brazil). Salvador sits on a vast bay, which at 1100 km, 70 km from north to south, and 60 km from east to west (at its widest point) is the largest in Brazil (Fig. 1). What appears to be the other side of the bay as you look out over the water from Salvador, is actually the Itaparica island. There are several ways of getting there: the ferry-boat system, the catamaran, by small boat, etc.

In this paper, we study the time series of vehicle demand between Salvador city and the Itaparica island by using the ferry-boat system [12]. This transport system has large economical importance and needs a lot of planning for its accomplishment.

2. Ferry-boat system of Salvador

The ferry-boat system transports vehicles and people from the continent to the island. A great number of tourists utilizes the ferry-boats, mainly in the summer and weekends. This fact leads to a complex system with a strong seasonal effect and weather influence (Fig. 2). As a general rule, the daily demand during the summer exceeds the daily demand during the winter (seasonal component).

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Fig. 1. The dashed line is the trajectory used by the ferry-boat system between Itaparica Island and Salvador city.

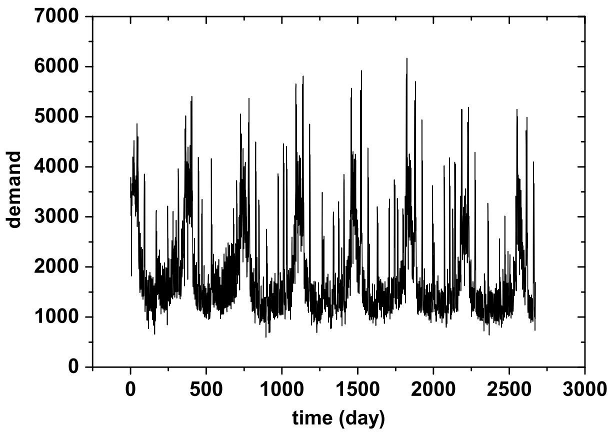

Fig. 2. The original time series of vehicle demand by using the ferry-boat system, these data correspond to demand collected between Jan. 1, 1996 and Apr. 26, 2003 daily.

3. Previous works

This seasonal component can be smoothed out by statistical methods. We compare here two distinct methods, namely, the density of mobile averages [13] and the DFA [14] with the usual statistical method (ARIMA model [15]). In a previous work [16], some of us show that these methods can be used to analyze this type of time series.

Self-affine signals characterized by Hust exponent can be investigated through mobile averages. The density of crossing points between any two averages (ρ) is a measure of long-range power-law correlations in the time series. The application of this technique was first made by Vandewalle and Ausloos [13] using an artificial time series (midpoint displacement). From the

$$\bar{y} \equiv \frac{1}{T}\sum_{i=0}^{T-1} y(t-i), \tag{1}$$

that defines the average of y for the last T data points we can build a classical mobile average. Considering two mobile averages $\bar{y_1}$ and $\bar{y_2}$ that are characterized respectively over T_1 and T_2. We apply this technique in a real time series of vehicle demand by using the ferry-boat system [16]. The two mobile averages used as T are data points $T_1 = 7$ (week) and $T_2 = 30$ (month).

Also we analyze this problem using Detrended Fluctuation Analysis (DFA) [14], a method proposed to analyze long-range power-law correlations in nonstationary systems. One advantage of the DFA method is that it accounts for the long-range power-law correlations in signals with embedded polynomial trends that can mask the true correlations in the fluctuations of a noise signal. The range of systems that apparently display power law, hence self-invariant correlations, has increased dramatically in recent years, ranging from base pair in DNA and its evolution [14,17], economics [18,19], climate temperature

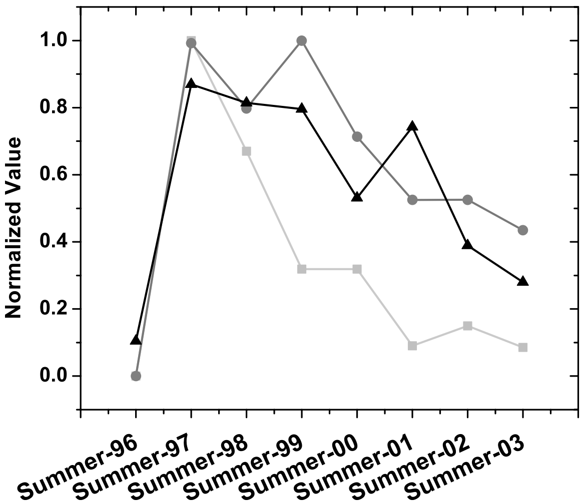

Fig. 3. The normalized data of vehicle demand by using classical statistics analysis (black triangle), DFA (gray circle) and density of mobile average (light gray square).

behavior [20,21], river flow and discharge [22,23], cardiac dynamics [24,25], light curves of astrophysical sources [26,27], phase transition [28], molecular dynamics [29] among others. The DFA method provides a relationship between $F(n)$ (root mean square fluctuation) and the box size n, characterized for a power-law $F(n) \propto n^{\alpha}$. In this way, α is the scaling exponent, a self-affinity parameter representing the long-range power-law correlation properties of the signal (fractal properties), such that if $\alpha = 0.50$ the signal is uncorrelated, if $\alpha < 0.50$ the correlation in the signal is antipersistent, and if $\alpha > 0.50$ the correlation in the signal is persistent.

In order to compare classical statistical analysis with density of mobile average and DFA methods we normalize all data, as showed in Fig. 3.

Fig. 3 depicts the main characteristic of all methods discussed here, using the classical statistical analysis (ARIMA method) or fractal analysis (DFA and density of mobile average). This figure shows that the vehicle demand decreases as the time increases. Here, the normalized version of the scaling exponent α seems to behave like to the normalized classical demand analysis and the normalized version of ρ falls faster than the other two methods.

4. Proposed model

We propose a simple model to evaluate vehicle demand dynamics. Our propose is that vehicle demand are build up like a diffusive process. Thus, the scaling exponent α indicates that the vehicle flux occurs in scale invariant media. Then this characteristic behavior strongly suggests that vehicle flux follows a Tsallis statistics [30,31], as showed in Ref. [32].

In recent years, Tsallis statistics has received increasing attention due its success in the description of certain phenomena exhibiting atypical thermodynamical features. For instance, the Tsallis formalism has been applied to protein folding [33–35], global optimization [36–38], stellar distributions [39], x-ray binary systems [40], dissipative systems [32], nonlinear Fokker–Planck equation [41,42] and many others [43]. Although this diversified application, Tsallis statistics was consolidated for complex systems analysis [43–47]. Commonly, Tsallis statistics covers this class of systems since it postulates a nonextensive (nonadditive) entropy S_q such that $S_q(A+B) = S_q(A) + S_q(B) + (1-q)S_q(A)S_q(B)$, where A and B are two independent systems in the sense that $P(x, x')_{A+B} = P(x)_A P(x')_B$. We recall that $P(x)$ is the probability density distribution of the macroscopic variable x, that characterizes the system. In this context, the so called entropic index q is a measure of the degree of nonextensivity. In the Boltzmann–Gibbs statistics, the entropy $S = -k \int P(x) \ln P(x) \mathrm{d}x$, gives rise to exponential probability density distributions, $P(x) \propto \exp(-x)$. Yet, within Tsallis statistics formalism, the maximization of the q-entropy

$$S_q = k \frac{\left(1 - \int [P(x)]^q \mathrm{d}x\right)}{(q-1)}. \tag{2}$$

Taking the vehicle flux (Fig. 1) as a dynamical process we can use a nonlinear Fokker–Planck equation to study this evolution. Then, we rewrite the equation in a convenient way,

$$\frac{\partial F}{\partial t} = -\frac{\partial}{\partial x}\{K(x)F\} + \frac{1}{2} Q \frac{\partial^2}{\partial t^2}\{F^{2-q}\} \tag{3}$$

where F is the distribution function of the vehicle demand, $K(x)$ the drift coefficient, Q the diffusion coefficient (a positive quantity) [41] and q is the entropic index.

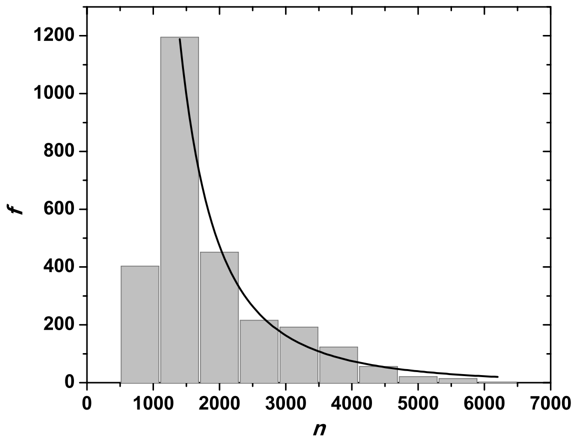

Fig. 4. The frequency (f) of vehicle demand (n) by using the ferry-boat system (gray bars). The black curve is the stationary solution of the nonlinear Fokker–Planck equation, i.e, a q-exponential curve.

Eq. (3), in the stationary solution,

$$\frac{\partial F}{\partial t} = 0 \tag{4}$$

presents the following solution:

$$F = A\left[1 - s_q\,(1-q)\,f(n)\right]^{1/(1-q)}, \tag{5}$$

i.e., a q-exponential as solution (Black line, Fig. 4), where

$$s_q = \frac{2}{Q}\left(\frac{A^{q-1}}{2-q}\right). \tag{6}$$

Fig. 4 presents the distribution of vehicle demand, i.e., its distribution depicts the frequency of number of vehicles (f). Thus, the frequency of number of vehicles follows a q-exponential distribution with $q = 1.39 \pm 0.32$, the correlation coefficient of Pearson $R = 0.99$ and $F_{value} = 350$ (Black line, Fig. 4). Therefore, the nonlinear Fokker–Planck equation is a candidate to explain qualitatively the vehicle demand (gray bars, Fig. 4).

5. Concluding remarks

In summary, we analyze vehicle demand by comparing two distinct methods, namely, the density of mobile averages [13] and the DFA [14] with the usual Statistical method (ARIMA method). From the analysis, it was found that these methods present a decreasing vehicle demand. On the other hand, the nonlinear Fokker–Planck equation can be used to obtain a qualitative behavior of the vehicle demand with a relevant statistical significance, when the number of vehicles is greater than $n = 1000$ vehicles/day.

Thus, this study can enable a contribution in terms of logistics and shipping, with respect to a plan that considers the improvement of internal processes, training of human resources, financial substantiality for investors, learning and growth, the ability to change and continuously improve the service provided. Both, exponent α and ρ point out scenarios of demand and Eq. (5) provides the average behavior of the vehicle demand. Then, these methods can be used in the demand analysis in general and can contribute with econometric analysis.

Concluding, we indicate the need for increased efforts to organize the local transport system, through coordination between the actors involved—state government of Bahia, the company responsible for the operation of the system and customers, for a better adaptation of the transport system to local and regional needs.

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