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Self-affinity and nonextensivity of sunspots

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ABSTRACT

In this paper we study the time series of sunspots by using two different approaches, analyzing its self-affine behavior and studying its distribution. The long-range correlation exponent α has been calculated via Detrended Fluctuation Analysis and the power law vanishes to values greater than 11 years. On the other hand, the distribution of the sunspots obeys a q-exponential decay that suggests a non-extensive behavior. This observed characteristic seems to take an alternative interpretation of the sunspots dynamics. The present findings suggest us to propose a dynamic model of sunspots formation based on a nonlinear Fokker–Planck equation. Therefore its dynamic process follows the generalized thermostatistical formalism.

The sun is essential to have life on Earth. A sunspot is an area on the photosphere that is marked by intense magnetic activity, which inhibits convection, forming areas of reduced surface temperature. They can be visible from Earth without the aid of a telescope. Besides, sunspot populations quickly rise and more slowly fall on an irregular cycle about every 11 years. The study of sunspots in the photosphere is still an interesting research field, as can be recognized in some recent papers on the properties of sunspot and related phenomena [1–3]. However, analyzing the time series of sunspots it can be noticed a noisy behavior that can be compared to other fluidic systems, i.e., the accretion power of stellar X-ray binary systems that are an important class of astrophysical objects [4–6] and the water liquid-vapor-phase transition [7].

The study of self-affinity of time series is long [8]. However some methodologies have been proposed for non-stationary time series with trends, such as detrended fluctuation analysis (DFA). Thus, DFA was proposed [9] to analyze long-range power-law correlations in nonstationary systems. One advantage of the DFA method is that it allows the long-range power-law correlations in signals with embedded polynomial trends that can mask the true correlations in the fluctuations of a noise signal. The DFA method has been applied to analyze DNA and its evolution [9,10], file editions in computer diskettes [11], economics [12], climate temperature behavior [13], phase transition [7], astrophysics sources [4,5] and cardiac dynamics [14,15], proteins [16], among others.

Fig. 1 depicts the scaling exponent α behavior. It is interesting to observe that the crossover behavior of the log–log plot of $f(t)$ versus time (t) for the sunspot time series is of $t \approx 135$ months. Therefore, in this case, the scaling exponent α shows the sunspot cycle. The upper inset shows that the data belong to a dataset collected by the Sunspot Index Data Center (SIDC) from 1749 up to the present [17]. If we use more elaborated methods like multifractal detrended fluctuation analysis (MF-DFA) [18] or Fourier detrended fluctuation analysis (FDFA) [19] scaling exponents can be obtained with a minor error, but we loose the crossover in the log–log plot of $f(t)$ as function of t. These three approaches allow power laws from the non-stationary time series of sunspots and anti-persistent scaling exponents can be observed [20].

The generalized thermostatistical formalism (GTS) proposed by Tsallis [21] has received increasing attention due to its success in the description of certain phenomena exhibiting atypical thermodynamical features. For example, GTS was consolidated for complex systems analysis [22–30]. But it was with regard to systems with long-range interactions that GTS has been consolidated. In fact, the first application of the Tsallis' ideas to a concrete physical problem was made by analyzing a type of astrophysical systems with long-range interactions called polytropic stellar distributions [31]. Within the conventional Boltzmann–Gibbs (BG) thermostatistics, the entropy additivity law, valid for extensive systems, is not appropriated for those submitted to long-range interactions as produced by gravitational forces. On the other hand, GTS covers this class of systems since it postulates a non-extensive (non-additive) entropy S_q such that

$$S_q(A+B) = S_q(A) + S_q(B) + (1-q)S_q(A)S_q(B), \tag{1}$$

where A and B are two independent systems in the sense that $P(x,x')_{A+B} = P(x)_A P(x')_B$. We recall that $P(x)$ is the probability

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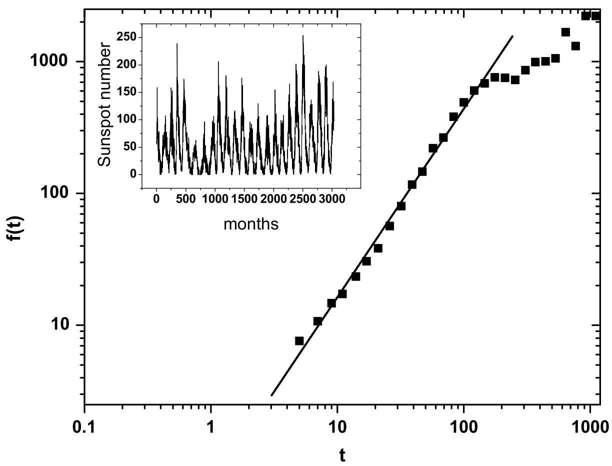

Fig. 1. The scaling exponent α for sunspot is $\alpha = 1.43 \pm 0.03$. The upper inset represents the number of these sunspots.

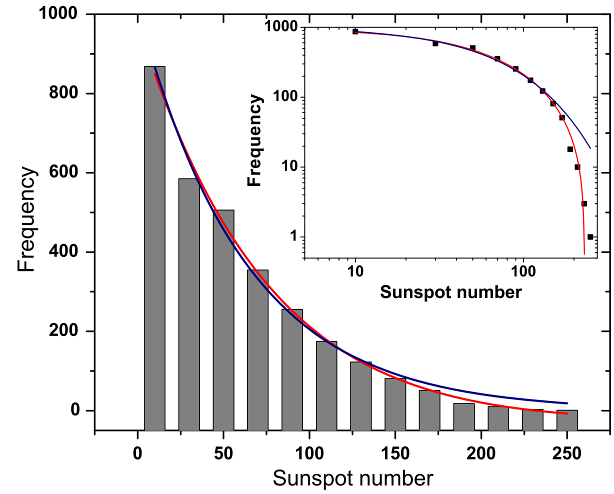

Fig. 2. The frequency of the number of sunspots. The q-exponential fitting (online red line) and exponential decay fitting. We recall that $q = 1.11 \pm 0.09$, the correlation coefficient of Pearson $R = 0.99$ and F-statistic value ≈ 2500 to the q-exponential fitting (red line). (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

density distribution of the macroscopic variable x, which characterizes the system. In this context, the so-called entropic index q is a measure of the degree of nonextensivity. In the BG statistics, the entropy $S = -k \int P(x) \ln P(x)\, dx$, gives rise to exponential probability density distributions, $P(x) \propto \exp(-x)$. Yet, within GTS formalism, the maximization of the q-entropy

$$S_q = k\left(1 - \int \left[P(x)\right]^q dx\right)/(q-1), \tag{2}$$

produces power-law distributions called q-exponential distributions, defined by:

$$P_q(u) \propto e_q^u \equiv \left[1 + (1-q)u\right]^{1/(1-q)}, \tag{3}$$

if $1 + (1-q)u \geqslant 0$ and $e_q^u = 0$ otherwise. In the limit $q \to 1$ the usual BG entropy is recovered, $S_1 \equiv S_{BG}$, and the q-exponential distribution converges to the usual exponential distribution.

The long-range gravitational forces form unique structures in the Universe called *self-gravitating systems* (SGS). Almost all the relevant structures such as stars, binary systems, galaxies, cluster or super-cluster of galaxies among others belong to this category. The nonextensivity of SGS is reflected in their long-range interactions. Moreover, there is another problem with the application of the BG thermostatistics to SGS: these structures have no absolute thermal equilibrium state, i.e. they are not isothermal displaying incomplete-equilibrium states characterized by time scaling properties. In fact, SGS never stops its evolution toward the absolute thermal equilibrium. Therefore, generalized thermostatistics formalism seems appropriate to study SGS since the q-exponential distributions [Eq. (3)] arising from the maximization of the q-entropy [Eq. (2)] are also solutions of the nonlinear Fokker–Planck equation [32] used to study systems evolving to thermal equilibrium. We remark that the usefulness of the Tsallis approach is also supported by their wide utilization in diverse astrophysical problems [33–35].

We propose a simple model to evaluate sunspots dynamics. Our proposal is show that sunspots are build up by a diffusive process. In this sense, as the sunspots time series presents a self-affine behavior and the solar system is a non-extensive system then a nonlinear Fokker–Planck equation is a natural candidate to evaluate this dynamic process.

$$\frac{\partial F}{\partial t} = -\frac{\partial}{\partial x}\left\{K(x)F\right\} + \frac{1}{2}Q\frac{\partial^2}{\partial x^2}\left\{F^\gamma\right\} \tag{4}$$

where F is the distribution function of sunspots, $K(x)$ the drift coefficient and Q the diffusion coefficient (a positive quantity) [32] and $\gamma \neq 1$ implies in a nonlinear solution to Eq. (4). In this sense, in the stationary solution, if we rewrite $\gamma = 2 - q$ and

$$\frac{\partial F}{\partial t} = 0. \tag{5}$$

Eq. (4) presents as solution the following q-exponential distribution:

$$F(x) = A\left[1 - \beta(1-q)V(x)\right]^{1/(1-q)} \tag{6}$$

where A is a positive integration constant.

Fig. 2 shows the distribution of sunspots, i.e., its distribution depicts the frequency of number of sunspots (f). Thus, the frequency of number of sunspots catalogued from 1749 until today follows a q-exponential distribution (online red line, Fig. 2) with a great statistical significance. We note that an exponential fitting (online blue line, Fig. 2) falls to $f \geqslant 150$ sunspots.

As the scaling exponent α indicates that the generation of sunspots occurs in scale invariant media and as the entropic index q is known to reflect fractality [36] these results (Fig. 1 and Fig. 2) strongly suggests that generation of sunspots follows a GTS. In this sense, the sunspots time series shows a dynamic evolution explained by the nonlinear Fokker–Planck equation, since the q-exponential behavior is obtained directly from stationary solution of this nonlinear equation [32].

In summary, sunspots generation obeys a complex and nonlinear diffusive process. Two complementary conclusions are observed here. First, the power law obtained from the sunspots time series vanishes to values greater than values close to 135 months. This small value (close to 11 years) is related to the cyclical behavior of the sunspots. This result seems to indicate a non-correlation between two consecutive sunspots cycles. Therefore, it seems unpredictable to say how sunspots are build up if we take into account time intervals greater than 135 months. On the other hand, the second conclusion obtained is related to the sunspots distribution. As a SGS the sun obeys a nonextensive behavior and its intense magnetic activity follows nonextensivity too. Thus, the nonlinear Fokker–Planck equation seems to be the actual solution for the diffusive process where sunspots are build up in the solar corona.

At least, the intense magnetic activity in the solar photosphere associated with the gravitational field turns the sunspots formation a nonlinear diffusive process. The sunspots distribution follows a

q-exponential decay that suggests a nonextensive behavior for the sunspots.

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References

[1] B.A. Ioshpa, V.N. Obridko, E.A. Rudenchik, Astron. Lett. 34 (2008) 210.
[2] M. Rempel, M. Schüssler, R.H. Cameron, M. Knölker, Science 325 (2009) 171.
[3] C. Fröhlich, Astron. Astrophys. 501 (2009) L27.
[4] M.A. Moret, G.F. Zebende, E. Nogueira, M.G. Pereira, Phys. Rev. E 68 (2003) 041104.
[5] G.F. Zebende, M.G. Pereira, E. Nogueira, M.A. Moret, Physica A 349 (2005) 452.
[6] M.A. Moret, V. de Senna, M.G. Pereira, G.F. Zebende, Int. J. Mod. Phys. C 17 (2006) 1597.
[7] G.F. Zebende, M.V.S. da Silva, A.C.P. Rosa, A.S. Alves, J.C.O. de Jesus, M.A. Moret, Physica A 342 (2004) 322.
[8] B.B. Mandelbrot, J. Bus. 36 (1963) 394;
B.B. Mandelbrot, The Fractal Geometry of Nature, W.H. Freeman and Co, New York, 1982.
[9] C.K. Peng, S.V. Buldyrev, S. Havlin, M. Simons, H.E. Stanley, A.L. Goldberger, Phys. Rev. E 49 (1994) 1685.
[10] C.-K. Peng, S.V. Buldyrev, A.L. Goldberger, S. Havlin, F. Sciortino, M. Simons, H.E. Stanley, Nature (London) 356 (1992) 168.
[11] G.F. Zebende, P.M.C. deOliveira, T.J.P. Penna, Phys. Rev. E 57 (1998) 3311.
[12] R.N. Mantegna, H.E. Stantey, Nature (London) 367 (1995) 46.
[13] P. Talkner, R.O. Weber, Phys. Rev. E 62 (2000) 150.
[14] P.Ch. Ivanov, M.G. Rosenblum, C.-K. Peng, J.E. Mietus, S. Havlin, H.E. Stanley, A.L. Goldberger, Nature (London) 383 (1996) 323.
[15] P.Ch. Ivanov, L.A. Nunes Amaral, A.L. Goldberger, S. Havlin, M.G. Rosenblum, Z.R. Struzik, H.E. Stanley, Nature (London) 399 (1999) 461.
[16] P.H. Figueirêdo, M.A. Moret, P.G. Pascutti, E. Nogueira Jr., S. Coutinho, Physica A 389 (2010) 2682.
[17] SIDC-team, Monthly Report on the International Sunspot Number, online catalogue, http://www.sidc.be/sunspot-data/.
[18] J. Kantelhardt, S.A. Zschiegner, E. Koscielny-Bunde, A. Bunde, S. Havlin, H.E. Stanley, Physica A 316 (2002) 87.
[19] C.V. Chianca, A. Ticona, T.J.P. Penna, Physica A 357 (2005) 447.
[20] M.S. Movahed, G.R. Jafari, F. Ghasemi, S. Rahvar, M.R.R. Tabar, J. Stat. Mech. (2006) P02003.
[21] C. Tsallis, J. Stat. Phys. 52 (1988) 479.
[22] C. Tsallis, Braz. J. Phys. 29 (1) (1999);
C. Tsallis, Braz. J. Phys. 39 (2009) 337.
[23] M.A. Moret, P.G. Pascutti, K.C. Mundim, P.M. Bisch, E. Nogueira, Phys. Rev. E 63 (2001) 020901(R);
M.A. Moret, P.M. Bisch, K.C. Mundim, P.G. Pascutti, Biophys. J. 82 (2002) 1123;
F.P. Agostini, D.D.O. Soares-Pinto, M.A. Moret, C. Osthoff, P.G. Pascutti, J. Comput. Chem. 27 (2006) 1142.
[24] M.A. Moret, P.G. Pascutti, P.M. Bisch, K.C. Mundim, J. Comput. Chem. 19 (1998) 647;
M.A. Moret, P.M. Bisch, F.M.C. Vieira, Phys. Rev. E 57 (1998) R2535.
[25] P. Douglas, S. Bergamini, F. Renzoni, Phys. Rev. Lett. 96 (2006) 110601.
[26] B. Liu, J. Goree, Phys. Rev. Lett. 100 (2008) 055003.
[27] R.G. DeVoe, Phys. Rev. Lett. 102 (2009) 063001.
[28] R.M. Pickup, R. Cywinski, C. Pappas, B. Farago, P. Fouquet, Phys. Rev. Lett. 102 (2009) 097202.
[29] M.A. Moret, V. de Senna, G.F. Zebende, P. Vaveliuk, Physica A 389 (2010) 854.
[30] A.J. daSilva, R.F. Lima, M.A. Moret, Phys. Rev. E 84 (2011) 041925.
[31] A.R. Plastino, A. Plastino, Phys. Lett. A 174 (1993) 384.
[32] A.R. Platino, A. Plastino, Physica A 222 (1995) 347;
B.M. Boghosian, Phys. Rev. E 53 (1996) 4754;
A.R. Platino, A. Plastino, Braz. J. Phys. 29 (1999) 79;
L. Borland, et al., Eur. Phys. J. B 12 (1999) 285;
C. Anteneodo, C. Tsallis, J. Math. Phys. 44 (2003) 5194;
A.R. Plastino, et al., Astrophys. Space Sci. 290 (2004) 275.
[33] C. Tsallis, D. Prato, A.R. Plastino, Astrophys. Space Sci. 290 (2004) 259, and references therein.
[34] C. Vignat, A. Plastino, A.R. Plastino, Physica A 390 (2011) 2491.
[35] A.C.P. Rosa, J.C.O. de Jesus, M.A. Moret, Physica A 392 (2013) 6079.
[36] M.L. Lyra, C. Tsallis, Phys. Rev. Lett. 80 (1998) 53.