Physica A 392 (2013) 6079–6083

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Nonextensivity and entropy of astrophysical sources

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HIGHLIGHTS

- We discuss the nonextensive behavior of astrophysical sources.
- The entropic index presents a universal behavior for different astrophysical sources.
- We show the relation between Tsallis entropy and entropic index (q).
- We observe a linear trend of Tsallis entropy and entropic index (q).

ARTICLE INFO

Article history:
Received 26 March 2013
Received in revised form 10 July 2013
Available online 25 July 2013

Keywords:
Tsallis statistics
Entropic index
Long-tail distributions

ABSTRACT

We study the X-ray intensities of 142 light curves of cataclysmic variables, galaxies, pulsars, supernova remnants and other X-ray sources present in the public data collected by the instrument All Sky Monitor on board the satellite Rossi X-ray Timing Explorer. We show that the X-ray light curves coming from astrophysical systems obey Tsallis's q-Gaussian distribution as probability density. This fact strongly suggests that these astrophysical systems behave in a non-extensive manner. Furthermore, the q entropic indices for these systems were obtained and they provide an indication of the nonextensivity degree of each of these astrophysical systems. The q-value increases for systems if the Tsallis entropy decreases.

X-rays arising from astrophysical systems are due to the diffusive processes that atomic particles are submitted to in their gravitation field [1,2]. Cataclysmic variables, X-ray binary systems, pulsars and quasars have their own characteristics, but a similarity shows up when we observe the power-law and scale-invariant correlations in their X-ray light curves [3,4]. In this sense, in recent years, there has been growing evidence indicating that astrophysical systems have self-affinity characterized by long range power-law correlations. In astrophysics, these long-range correlations have been applied to the study of simulations in self-gravitating systems—non-equilibrium structures that feature characteristics of long-range correlations [5]—in the analysis of galaxy distribution [6] and in the characterization of self-similarity in solar active regions [7]. For X-ray binary systems [3,8,9] we observe self-affinity in their light curves, where the accretion mass rate in the accretion disk defines their scaling invariance. For all types of astrophysical sources (e.g. X-ray binary systems, cataclysmic variables, pulsars, galaxies, etc.) a universal behavior regarding self-affinity is observed [4]. The main purpose of this paper is to study the properties of the intrinsic distribution of light curves obtained from X-rays coming from different astrophysical sources.

It is clear that the statements about the nonuniversality of the microscopic expression for the entropy are by no means self-evident [10–12]. It is through a variety of tests that we have come to such a possibility. The Tsallis formalism has been applied to a variety of problems such as global optimization [13–16], protein folding [17–23] and the quantum mechanics

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http://dx.doi.org/10.1016/j.physa.2013.07.034

formalism [24], among others. Thus, the Tsallis entropy

$$S_q = k\frac{1-\sum_i P_i^q}{q-1} \tag{1}$$

is a physically meaningful generalization of the Boltzmann–Gibbs statistics, and when $q \to 1$ we recover the Shannon entropy ($S = -k\sum_i P_i \ln P_i$).

On the other hand, our main interest is the non-extensive behavior in astrophysical [25–33] and cosmological [34,35] self-gravitating systems. However, the astrophysical nonextensivity is a rather complicated problem to tackle from a theoretical point of view. In some sense, this is something to be expected given the long range of the gravitational interaction. Our present aim is to remind the reader that in astrophysics, within the standard thermodynamical approaches, it is not possible to simultaneously have finite values for the total energy, entropy and mass of a self-gravitating system.

We investigate self-gravitating systems from a non-extensive point of view. The non-extensive behavior takes the following entropy relation:

$$S_q(A \oplus B) = S_q(A) + S_q(B) + (1-q)S_q(A)S_q(B) \tag{2}$$

where S_q is the entropy and the entropic index $q \neq 1$ defines the non-extensive behavior as well as characterizing the degree of nonextensivity (Eq. (2)).

In this paper, we determine that the above systems obey a generalized statistics and therefore they can be interpreted as non-extensive. In addition, we study their degree of nonextensivity, analyzing their light curve distributions.

The data used in this paper are a product of the observations made by the instrument All Sky Monitor (ASM), on board the scientific satellite Rossi X-ray Timing Explorer (RXTE) to study the X-ray sky. The ASM–RXTE has been operational since 1995 covering a spectral band of X-ray photons corresponding to 1.5–12 keV [36]. The main goal of the ASM is to alert observers to the appearance of transient X-ray emitters and a long-term monitoring of bright X-ray sources. The ASM consists of three scanning shadow cameras (SSCs) mounted on a motorized rotation drive. Each SSC contains a position-sensitive proportional counter that views the sky through a slit mask. Event data are normally compressed within the two ASM event analyzers in the experimental data system and relayed to the spacecraft for insertion into the telemetry stream. One ASM event analyzer accumulates histograms of counts binned as a function of position. These position histograms are accumulated in series of 190 "dwells". During each dwell, the spacecraft maintains a fixed attitude and the ASM rotation drive, which is also controlled by this event analyzer, is not active, so that the orientation of each SSC is fixed in relation to the sky. A position histogram thus contains the superposition of the mask shadows from each X-ray source in the field of view during a single dwell. The other ASM event analyzer produces count rates for both X-ray and background events.

The ASM data analysis proceeds by computing intensities for sources listed as active in a master catalog and then searching for and locating additional sources. Source intensities are obtained from the solution of a linear least-squares fit of position histograms, with model shadow patterns for each active source within the field of view and with patterns representing non X-ray and diffuse X-ray backgrounds [37]. The fitted solution also yields estimates of the uncertainties of the derived intensities; these uncertainties are based purely on the photon counting statistics predicted by the best-fit model. Fitted intensities are normalized to on-axis count rates in SSC 1. This requires corrections, so that the results from all three SSCs are consistent. Source intensities and errors are obtained from the fit with the minimum value of the reduced χ^2 statistic.

We looked into the intensities of 142 X-ray light curves of the aforementioned astrophysical X-ray sources. We examined the frequency distribution of light intensity of the following sources: cataclysmic variables, Seyfert galaxies, quasars, BL Lacertae objects, normal and active galaxies, supernova remnants and pulsars, present in the public data collected by the instrument All Sky Monitor. Similar kinds of astrophysical systems were grouped into different datasets to be analyzed independently.

The first dataset refers to cataclysmic variables (CV), which include all those binaries in which a red dwarf (or sub dwarf) star and a white dwarf mutually interact.

The second dataset we analyzed was Seyfert galaxies, quasars, BL Lacertae objects, normal and active galaxies. Seyfert galaxies are predominantly spiral galaxies that have a very bright, hot, star-like nucleus, which emits non-thermal radiation and is brighter than the galaxy itself. The quasar phenomenon is probably just an extreme form of what is found in Seyfert galaxies. It should be noted that quasars have the highest absolute luminosities of any of the known objects in the universe. The BL Lacertae objects (third dataset) differ from typical quasars mainly in their continuous spectrum [38]. This group of objects obeys the q-Gaussian distribution for all analyzed sources as also does our fourth dataset, supernova remnants.

The fifth dataset is for the pulsars (the name derives from pulsating radio stars). We recall that the observed cyclic variability in the radio and X-ray emission from these objects is not caused by pulsation in the star, but rather by their rapid rotation. The sample follows the q-Gaussian behavior.

The shapes of the intensity distribution plots for all astrophysical sources roughly follow q-Gaussian distributions. This is interesting because all these types of X-ray sources behave in the same way regarding the Tsallis distribution.

It is interesting to note that, overall, most of the observed systems seem to comply with Tsallis distributions because this may be due to the fact that the distributions that better describe the intensity of the X-ray light curves have very long tails [39]. We recall that the distribution plots of all types of astrophysical sources follow this behavior. We also recall that other astrophysical phenomena like X-ray binary systems, small open clusters, globular clusters and close binary stars also present long tail distributions. Thus, these are candidates to follow a non-extensive statistics behavior too.

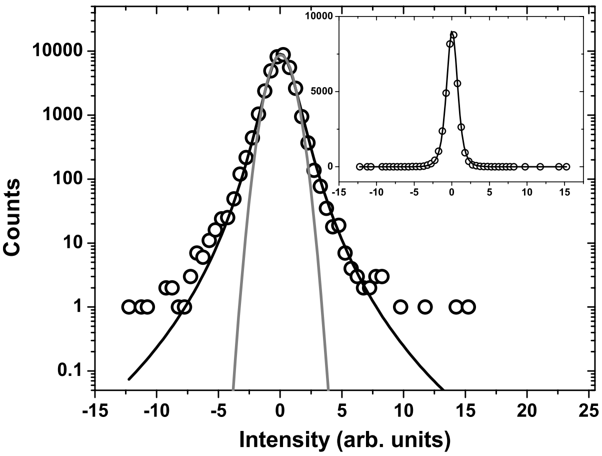

Fig. 1. The q-Gaussian fitting ($q = 1.34$) of the Puppis A system which is a supernova remnant. Black circles represent the histogram of the Puppis A X-ray intensity, the black line is the q-Gaussian fitting and the gray line is the Gaussian fitting.

As was previously mentioned, a diffusive process drives the X-ray generation of the astrophysical sources. Hence, a Tsallis entropy (Eq. (1)) must solve this type of astrophysical/cosmological problem [12]. Therefore the probability distribution of the Tsallis entropy

$$P_q(x) = A_q e_q^{-B_q(x-x_0)^2}, \tag{3}$$

where

$$e_q^y \equiv [1 + (1-q)y]^{\frac{1}{1-q}}, \tag{4}$$

must give a correct fitting to the astrophysical X-ray intensity. The proposed solution (Eq. (3)) is exactly the Gaussian-curve generalization, which has been used in different contexts [9,12,24,33]. Fig. 1 shows a typical q-Gaussian fitting.

The q-Gaussian fitting (Fig. 1) seems to be the best one for astrophysical sources. For the Puppis A system the q-Gaussian fitting presents a Pearson correlation coefficient of $R = 0.999$, $F_{\text{Value}} > 20{,}000$ and Prob $> F_{\text{Value}} = 0$ (black line). Also we observe that the gray line (the Gaussian fitting) vanishes and does not follow the observed X-ray values greater than 1 standard deviation. On the other hand, the black line (q-Gaussian fitting) fits the observational values well. We recall that the choice of the Puppis A system, a supernova remnant, was a random choice. All other sources were clearly fitted by q-Gaussian distributions. Since the entropic index q is known to reflect fractality [40,41] the present result strongly indicates that the generation of X-ray occurs in scale invariant media, as observed in Refs. [3,4,8].

Fig. 2 depicts the mean value of entropic index (q) for different astrophysical sources. From this figure we notice that these X-ray intensity curves present a clear and universal behavior of the entropic index. The analysis (Fig. 2) indicates that for supernova remnants the q-value is $\langle q\rangle = 1.42 \pm 0.06$, $\langle q\rangle = 1.42 \pm 0.05$ (galaxies), $\langle q\rangle = 1.42 \pm 0.05$ (cataclysmic variables), $\langle q\rangle = 1.42 \pm 0.05$ (blasars) and $\langle q\rangle = 1.41 \pm 0.05$ (pulsars). We recall that the emerging energy from these systems is obviously a non-extensive energy and therefore the self-gravitating interactions are long range interactions.

This universal behavior seems to be independent of how it originated X-rays in the different astrophysical systems analyzed. Also, we note that these light curves present a clear and universal self-affine behavior [4]. In fact this universality by itself does not give a relevant indicator to distinguish the astrophysical sources. Moreover, an interesting behavior is observed when we analyze the q-entropies of different astrophysical systems as a function of q entropic index, as shown in Fig. 3. The graph shows a linear trend which justifies the linear fit where the coefficients assume values of $A = -1,42\pm 0,03$ and $B = 3,30 \pm 0,05$. The correlation coefficient equals 0.94. This trend can be modeled if we apply an expansion in power series of S_q (see Eq. (2)).

In fact, simply replacing $P_i^q \to P_i \mathrm{e}^{(q-1)\ln(P_i)}$ in Eq. (2) allows us to develop the exponential function as a power series of $(q-1)$. After simple calculations and using the normalization condition $\sum_i P_i = 1$, we get a linear expression of S_q as a function of q in the form $S_q \approx Aq + B$, where the coefficients are given by

$$A = -k\left(\frac{\sum P_i[\ln(P_i)]^2}{2}\right) \tag{5}$$

$$B = k\left[\left(\frac{\sum P_i[\ln(P_i)]^2}{2}\right) - \sum P_i \ln(P_i)\right]. \tag{6}$$

Clearly, the limit $q \to 1$ leads to Boltzmann–Gibbs statistics. This result supports the linear trend shown in Fig. 3.

In summary, we investigate the nonextensivity of the X-ray light curves coming from astrophysical objects and observe q-Gaussian distributions (i.e., long-tail distributions) in these X-ray light curves. In the narrowest sense, we observe that the

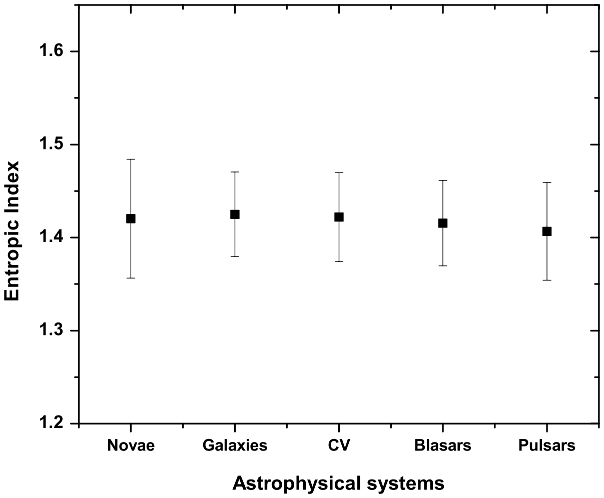

Fig. 2. The expected q value for supernova remnants (Novae), galaxies, cataclysmic variables (CV), blasars and pulsars.

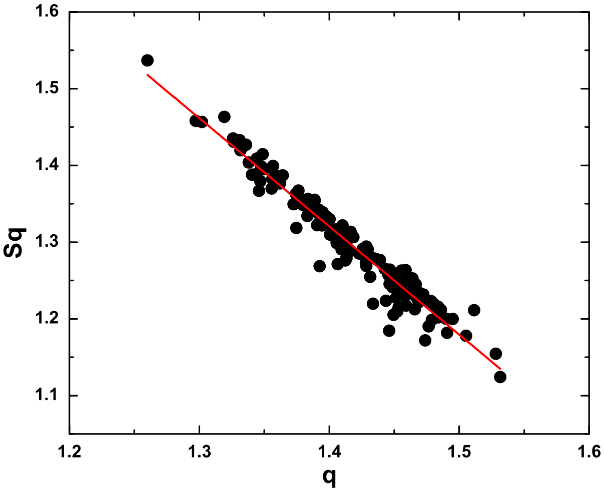

Fig. 3. The Tsallis entropy values (S_q) as function of the entropic index (q) for all 142 light curves of astrophysics sources.

q entropic index has a universal behavior and it presents $q = 1.418 \pm 0.007$ as its expected value. We recall that despite the different ways of producing X-rays, non-extensive behavior remains universal. Therefore, the long-tail distributions in X-ray time series produce a signature in these time series. We recall that the q-Gaussian distributions observed from the astrophysical sources analyzed in this paper are due to self-gravity field in these systems. Then, these nonextensive distributions have different backgrounds of non-Gaussianity observed in the cosmic inflation, i.e., the primordial origin of cosmological fluctuations that was a period of accelerated expansion at very early times [42,43].

In relation to the Tsallis entropy (S_q) as a function of the q entropic index, it was shown that the these astrophysical objects follow a linear trend, i.e., $S_q \approx Aq + B$. This result suggests that if the system is more entropic, the value of the associated q entropic index will be lower and the nonextensivity of the system will be lower.

Acknowledgments

We wish to thank the CNPq and FAPESB (Brazilian grant agencies) for financial support.

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